SPECIAL ARTICLES.

APPLICATION OF MATHEMATICS IN METEOROLOGY.

By Prof. FRANK HAGAR BIGELOW.

There are three processes that are generally essential for the complete development of any branch of science, and they must be accurately applied before the subject can be considered to be satisfactorily explained. The first is the discovery of a mathematical analysis, the second is the discussion of numerous observations, and the third is a correct application of the mathematics to the observations, including a demonstration that these are in agreement. As a matter of fact, however, the history of science shows that there has been great difficulty in the course of the working out of new problems to bring this about satisfactorily. Sometimes the theory is in advance of the observations, as in Maxwell's theory of electricity and magnetism. Again, the observations are in advance of the theory, as in solar physics and terrestrial magnetism. Often a good theory is misapplied to good observations, or good observations are explained by a poor theory. Whenever any of these unfortunate procedures take place the progress of science is retarded. When a good theory is misapplied, there must follow it a searching criticism and all things must be rebuilt from the foundations. Thus, the Ptolemaic cycles and epicycles were good as theory for a geocentric solar system, but the Copernican and the Newtonian theories could be applied to Kepler's observations only by a complete destruction of the ancient astronomy.

There is perhaps no branch of modern science that has suffered more severely than meteorology by the misapplication of good mathematics to good observational data. Of course in this case, as in other instances, the observations, while good so far as they went, did not sufficiently cover the field of research, so that it was possible to propagate theories which apparently harmonized with the observations then at hand. Thus, for more than half a century the atmospheric observations were confined to the surface of the earth or to the very lowest layers of the air. It is only within ten years that the upper-air observations have been made in sufficient numbers to fix our attention upon the true circulation of the great currents of air in the general cyclone and in the local cyclones and anticyclones. The data obtained by the cloud computations, or by the balloon and kite ascensions, have made it possible to examine critically the existing theories, with the unfortunate result that nearly the entire range of general theory of the circulation of the atmosphere must be pronounced a misfit. Had the modern observations been in the hands of Professor Ferrel or Professor Oberbeck, it is hardly possible that they would have written as they did. Indeed, there are probably very few scientific theories which have had a wide acceptance against which such grave and intractable objections exist as against the vertical convection theory of the origin of storms by Espy, which derives the source of the energy expended in cyclones from the local condensation of aqueous vapor and the setting free of the equivalent latent heat. It is not my purpose to review this subject in detail, as that has been done elsewhere, but I wish to summarize the mathematical state of the problem in a few words and to indicate the direction in which the great theories of meteorology will probably be reconstructed.

LOCAL CYCLONES AND ANTICYCLONES.

(1) Espy-Ferrel theory.—Speaking generally, the problem of cyclones has been treated as independent of that of the general circulation. Local sources of heat, forming ascending central columns of air have been assumed, as the generating energy of the cyclonic vortices, quite apart from the great excess of temperature in the tropical zones which produces the general vortex covering a hemisphere. Ferrel assumed that the hemispherical vortex and the local vortex are similar in structure, but quite independent of each other in the sources of their energy. He drew a bounding surface around a mass of air, warm or cold at its center, as the case might be, and discussed the resulting vortex. The laboratory confirmed the derived circulation by placing water in a cylindrical vessel rotating as a whole about a vertical axis, the central portions being heated or cooled, or else having a vertical central current produced mechanically. There was no mathematical objection to the Ferrel vortex itself, nor to the laboratory experiment, until it was attempted to match these results with the observed atmospheric facts. Meteorologists who were in any ways critical have found such difficulty in accounting for the local supply of central heat as to be quite doubtful about the value of Espy's source of energy, and this source was reluctantly adopted by Ferrel himself. The Weather Bureau observations of 1896–97 traced out the stream lines of circulation with sufficient exactness to terminate this part of the discussion, by showing that in the local cyclones and anticyclones the air does not circulate as the Ferrel vortex requires. Hence, we conclude that Ferrel's application of mathematical analysis to the explanation of cyclonic observations is not satisfactory. This eliminates a long section from the literature of meteorology.

(2) The German school of meteorologists, Reye, Mohn, Sprung, Oberbeck, began with another type of vortex motion, having also a beautiful mathematical analysis, depending upon a local overheated central column. Here, again, the objections are prohibitory, first as to the origin of the cyclonic heat for vertical convection, and, second, as to the nonconformity of the observed stream lines with the theoretical vortex.

These two types of vortices are entirely distinct from each other: Ferrel's has a cylindrical bounding surface, a zero velocity where the direction of gyration vanishes, and stream lines continuous within the same mass of liquid; the German vortex has no outer boundary, but a circumscribed inner region with vertical velocity increasing as the height, an outer region with no vertical component, and a maximum velocity at the separation of these two regions. One may frequently observe the German vortex in snow or dust whirls, when the currents of air are sharply deflected by walls and adjacent buildings. It is necessary, therefore, also to exclude the German vortex from modern meteorology, and this removes another large chapter from its literature. Fortunately, the treatment of the tornado vortices has been substantially correct, but meteorology must make a fresh start with the theory of cyclones and anticyclones. A series of suggestions can be found on this subject in my recent reports as to the kind of work required; but the task is one of great difficulty, and it may require much labor to finish it.

THE GENERAL CYCLONE.

The history of the theory of the general cyclone is very similar to that of the local cyclone. There exist two great analytic discussions, Ferrel's and Oberbeck's, and, while they have much in common, the results partially contradict each other and they are only in partial agreement with the observations. In the case of the general circulation the analysis and the observed conditions harmonize better than in that of the local cyclone, and it is therefore necessary simply to improve the details of the analysis, although the general circulation is really not so simple as is called for by that theory. To illustrate briefly, Ferrel derives a very powerful eastward movement at the poles and a vanishing motion at the equator, while Oberbeck reverses this conclusion. Ferrel and Oberbeck make a powerful northward component in the upper strata of the atmosphere and a strong southward component at the surface; but observations show that a very feeble poleward component is flowing in the upper air, and that in the

lower air a series of irregular currents pass each other on the same levels instead of above one another at different levels. The canal theory found a laboratory experiment to match it by heating water at one end of a long box, when the resulting motion apparently satisfied the mathematical analysis, though that was before the international observations were made.

My reconstruction process is quite simple in conception, but intricate in its details. For the local vertical central convection current is substituted a general system of horizontal currents flowing from the Tropics and polar zones, respectively, into the middle zones. For the general canal theory of the overflowing strata is substituted a counterflow of currents in the lower strata, and on the same levels. The cyclones and anticyclones are due to the interaction of these horizontal currents of air of different temperatures, which transport the enormous energy derived from the solar radiation in the Tropics, and expend it in raising the air in the polar zones to a higher temperature, the cyclones being the mechanical products of this thermodynamic process. The observed stream lines and the computed isobars in the higher levels point to this view in the most positive manner, and it is itself in harmony with the requirements of thermodynamics as well as hydrodynamics, assuming a type of engine which is constituted like that of an atmosphere heated in the equatorial regions of a rotating globe. Unfortunately for meteorology, this statement shows that it is now necessary to reconstruct a great portion of the old theory of the general cyclone, and to reject entirely the theories which have been proposed to explain local cyclones and anticyclones.

The reversal of important scientific researches by the progress of investigation is so common in the history of science, that it brings no discredit upon students who have explained matters according to the data in their possession. Indeed, Lord Kelvin considers it to be a "point of honor" to make such reversals for one's self in the interests of perfect scientific truth, and he has set the world an illustrious example of this high-minded candor and self-effacement. Thus, a half century ago he regarded the ether as subject to gravitation, but he now treats it as a substance outside the law of gravitation. He developed his famous theory of the atoms consisting of the ether isolated in dynamic vortices, but now regards this idea as untenable and has taken up again the old Lucretian mass atom as most likely to prove correct. In his presidential address of 1893 he gave an example to show that the sun and the earth have no causal magnetic connection, but it is understood that he now thinks that the observed synchronism between the variations of the solar faculæ, spots, and prominences on the one hand, and the elements of the terrestrial magnetic field on the other, is so persistent as to make it necessary to reverse that conclusion. The influence of an apparently valid result of the discussion of observations by a scientist of undoubted ability, fortified by a powerful mathematical treatment, sometimes turns aside the advance of knowledge into a wrong path, and this may even stop for a time all further efforts to solve the problem. Such failures, of course, should be reckoned as only the profit and loss in the bookkeeping of research, and such temporary checks must not be taken too seriously.

THE THEORY OF LEAST SQUARES IN METEOROLOGY.

Professor Schuster has recently urged upon meteorologists the importance of submitting their researches to the analysis of the Fourier series, and the theory of least squares, in order to test properly the periodicities derived from the observations, and he has illustrated his views by applying his periodogram or probability curve to check the various periods that have been derived for the solar rotation. Fourier's Theorem has already been widely used by meteorologists to express many of the periodic functions observed in the atmosphere, and some prefer this method to the numerical or the graphic methods, in spite of its great additional labor. Astronomers and physicists have used the probability theorem freely and with valuable results, but only in certain restricted classes of observations. This theorem requires that the events shall be *independent of each other*, and if this criterion fails, then the entire process is invalid. Thus, the independent observations on a star's place, the separate measures of a physical quantity in the laboratory, and such like matters, may be tested for probable accuracy by this method. The distribution of all the waves emitted by a black body at a given temperature T being according to the law of errors, this may be computed as a probability curve, since the normal energy gives a spectrum curve with lines of variable intensity for the several independent wave lengths. In the kinetic theory of gases, the several independent velocities which inhere in the moving molecules may be tested for their respective intensities when the total kinetic energy of the mass is known.

Suppose, now, one proposes to apply that theorem to the events recurring in the circulation of the earth's atmosphere such as the temperature changes, the variations of the pressure in cyclones, or the observed conditions of the aqueous vapor as vapor tension or precipitation at a given station. What reason is there to assume that these elements as they occur from day to day are independent of each other? The pressure, temperature, and vapor tension at a station are in fact the results of a very complex circulation which passes over a station as the effect of conflicting currents flowing from the polar and tropical zones, due to the incessant struggle of the elements toward equilibrium in this thermodynamic medium. We have to deal with no single system of independent events, as the waves in a normal energy spectrum, or the molecular velocities in a gas of given energy, but there are many series of interdependent events inextricably interwoven. It is seldom that the meteorologist has a pure series of events to work with, as the astronomer or the physicist has in many of his observations, and that is why the meteorologist has a peculiarly difficult task, and why his science is not yet perfected. Nevertheless, these problems are most fascinating, and they will probably in the future engage more of the attention of astrophysicists and of mathematical physicists, because they afford concrete examples of the most profound questions in theoretical physics.

THE THEORY OF LEAST SQUARES IN SOLAR PHYSICS.

When we come to solar physics the case is even more troublesome. There we have at an enormous distance from us an immense mass of seething matter at very high temperatures. From observations on the surface phenomena of the sun, the inference is inevitable that all the intractable conditions which on the earth render it difficult to apply the probability theorem are there multiplied in their complexity. The recurrences of the spots, faculæ, and prominences on the surface of the sun are simply resulting products of very complex processes going on in the interior, and in the circulation attendant upon the readjustments of its thermal equilibrium. If the solar radiation falling on the earth's tropical zones produces the observed complex circulation of the atmosphere with its interpendent current systems, how much more should this be true in the sun's circulation. By so much more will it be impracticable to apply correctly the least-square theorems or the potential theorem, as Professor Schuster has attempted to do in various ways. We must for the present, until the true nature of the physical problem is understood, approach the solution by more simple, practical methods. It only paralyzes the efforts of students to have negative results derived from mathematical analysis laid down as decisive, and the only effect is to hinder such advances as can probably be made by the simpler graphical or numerical methods.

Take, for example, the rotation period of the sun, which has been determined many times from the recurrence of various ter-

restrial and solar events. On the surface of the sun we observe a synodic period of 26.68 days at the equator, a longer period of 27.30 days at latitude 12°, and still longer periods at higher latitudes approaching 29.50 days near the poles. There is evidence that the period varies also with altitude as well as with latitude. Now, several periods computed in the terrestrial field have been announced to be about 26.00 days—that is, three-fourths of a day shorter than the period observed at the solar equator, which is the smallest period that can be seen on the surface of the sun. Is it probable that at the distance of the earth the angular velocity is much larger than the greatest visible in any part of the photosphere? We may note in regard to the several discussions of this subject that the motion of the atmosphere relative to the surface of the earth, which carries with it the thunderstorms, the aurora, and the electric potential, has not been eliminated from the computed periods. This should be done, and it would result in lengthening the 26.00-day period. Several of these solutions have been executed by least-square methods in one form or another, and the fact that there has been a general failure to come to any agreement as to the true period of the sun's rotation influenced me to employ a simple computation and tabular exhibit of the facts, which would exhibit the periodic events as they occur. On laying down the azimuth angles of the deflecting vectors of the earth's magnetic field in long tables, a marked periodic phenomena became evident, and it persisted through the series of fifteen years over which the work was extended. Now, while it was easy to note the general features of this periodic action and to mark the dates of transition in azimuth, the periodic recurrence was attended in general by an irregular sliding backward and forward through short intervals on both sides of the mean, causing a lap of a day or two on each side of the average periodic time. The actual dates were marked down; an approximate period and epoch were assumed; the system of residuals was determined between the observed and computed dates, and then the adjustment of the assumed period and epoch was made by least squares. It is undoubtedly proper to apply least squares to these data. This unsteady action in the 26.68-day period is like that occurring in the 11-year sun-spot period, which has similar irregularities, some individual periods being longer and some shorter than the average, but from these one can compute the mean period, as Professor Newcomb has recently done by the same least-square process. Now, in the case of the resulting 26.68-day rotation-period there is a further complexity to be considered. The intensity curve is not simple, but it is one having several crests about three days apart, and this shows that the solar output is very unsteady in longitude as well as in latitude. If this curve is developed quite loosely in longitude and the crests move back and forth, as is natural in such a congested struggling medium, then there is a tendency for the crests of the curve in one period to fall upon the corresponding hollows in another period, and thus the maxima and the minima neutralize each other. The result of this fluctuating action is that there is an excessive waste in the summation of the numerical matter, whether by the graphic or the periodogram methods, and the inference that no average period exists is a misapplication of the logical conclusions that should be made. If, then, a fixed period is adopted, and the least-square theorems are rigidly applied as if the events were simply independent and recurring at random, a negative result will certainly be obtained.

Hence, it is evident that one should be very cautious in the application of mathematical analysis to the observations of solar physics generally, and, without such caution, negative results will have very little critical value. It may be well to point out in this connection that the 11-year period of solar-spot formation is confined to the middle latitudes of the sun, from +35° to —35°, and that both polar regions are quite free from this special periodic phenomenon. This result was obtained from the discussion of the Italian observations on the solar prominences, which in the middle zones have the same 11-year period as the spots and the faculæ, but do not continue with this period into the polar latitudes. That fact suggests that too much emphasis may have been laid upon the 11-year synchronism in discussing these solar-terrestrial problems. On the other hand, I have found a 3-year cyclic recurrence which is more characteristic of the entire surface of the sun, and this short cycle has been shown to exist simultaneously in the terrestrial magnetic field, also, in the pressure and temperature variations, and hence in the circulation of the atmosphere generally. It is quite likely that we shall find in this short cycle more evidence of synchronism between solar and terrestrial events than in any other period that has been examined.

In conclusion, we may observe that profound mathematical analysis does not guarantee that the simple law inherent in the physical conditions observed has been secured. There are enough failures of that kind to make one suspicious, because it often happens that the mathematical symbolic language of the equations obscures the implied thought, which is in itself simple, such as might first be brought out by graphical methods. Also, it is evident that negative results have very inferior weight when they proceed from intricate discussions, if the observations naturally bear another interpretation, for the unsuspected secrets of nature still contain surprises to man's inquiring reason.

TORNADO IN EASTERN ALABAMA, MARCH 20, 1905.

By FRANK P. CHAFFEE, Section Director, Montgomery, Ala.

The tornado was first felt about 6:20 p. m. (seventy-fifth meridian time) of the 20th, at Doublehead, in the northern portion of Chambers County, where one frame building was demolished, one person killed, and two severely injured. The storm crossed the track of the Central of Georgia Railway about two miles north of Welsh, near a settlement known as Bacon Level, where several frame houses were destroyed and four persons seriously injured. A few miles farther east, on Wilson's Plantation, ten people were seriously injured and one frame house demolished; on Holley's Plantation, in the same vicinity, a frame house was blown down and an entire family, consisting of seven persons, was killed and two persons were seriously injured. From this point the storm curved northward to Lime, Randolph County, where several frame buildings were destroyed, and two persons were fatally injured. The storm then passed off northeast into Heard County, Georgia.

The tornado occurred in the southeast quadrant of a general storm eddy, which moved northeastward across northern Alabama on the afternoon of the 20th. It lasted but a few minutes; its path, which extended from southwest to northeast, was about eighteen miles long and varied in width from 75 to 200 yards. It is reported that a well-defined funnel-shaped cloud was observed, which had a bounding motion and which seemed to contract as it struck the ground at points of greatest destruction, the cloud swelling each time it left the ground. A crackling, rumbling noise was heard from the cloud, around which bright, but not particularly vivid, lightning played. In the center of the path débris was carried forward, while on the outer edges much of it was carried in the opposite direction. The funnel-shaped cloud was very dark, and was accompanied by a heavy downpour of rain, the latter lasting about ten minutes.

At Montgomery, about 72 miles southeast of where the tornado started, warm, unsettled weather prevailed during the afternoon of the 20th, with a maximum wind velocity of 22 miles per hour from the southwest.

Total number of persons killed along the storm's path, 9; fatally injured, 2; seriously injured, 18; estimated damage to buildings, timber, and fences, $5000.